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CLOSURE OF A CAVITY IN POLYMER LIQUID

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The closure of a spherical cavity in a relaxing polymer liquid with nonlinear rheological equations is investigated.

Known experimental results indicate anomalous dynamics of bubbles in liquids containing polymer additives. As well as the integral effect of suppression of cavitation of various types in solutions of high-molecular compounds [1], slowing of the collapse of individual inclusions has been noted [2], together with stabilization of the spherical form and retardation of the development of microjets in the closure of bubbles close to solid boundaries [3]. The theoretical model of cavity growth and collapse in a polymer medium was formulated in [4, 5], respectively, within the framework of a spherically symmetric formulation of the problem. To describe the rheology of the liquid, the Oldroid equation with an upper convective derivative was used [6]. The equations of gas-bubble oscillation in this liquid were obtained in [7]. Note that nonlinear pulsations of the bubbles in a viscoelastic liquid were also numerically investigated in [8-10], but instead of the corresponding invariant time derivative the ordinary derivative d/dt was used for the tensor quantities in [8, 9] and the partial derivative $\partial/\partial t$ in [10].

Numerical calculations of the nonlinear dynamics of a cavity in a relaxing polymer medium on the basis of integrodifferential equations [4, 5] are sufficiently difficult (in [5], because of the development of numerical instabilities, only the initial stage of collapse was calculated), which complicates the use of such equations, in particular, for the description of collective phenomena. It is shown below that integrodifferential equations of the type in [4, 5] may be reduced to equivalent differential equations, and on this basis the features of cavity closure in polymer liquid are analyzed.

The equation of radial bubble motion in an incompressible non-Newtonian liquid takes the form

$$I = p_\infty - p_g + 2\sigma R^{-1} = S, \quad I = \rho\left(R\ddot{R} + \frac{3}{2}\dot{R}^2\right),$$

$$S = 2\int_0^\infty (T_{rr} - T_{\varphi\varphi})(3y + R^3)^{-1}\,dy. \tag{1}$$

For T, a rheological equation of Oldroid type is adopted [6]

$$T = T^{(1)} + T^{(2)}, \quad T^{(2)} = 2\eta(1-\beta)D, \tag{2}$$

$$T^{(1)} + \lambda\left[DT^{(1)}/Dt - \alpha(T^{(1)}\cdot D + D\cdot T^{(1)})\right] = 2\eta\beta D.$$

When $1/2 < \alpha \leq 1$, Eq. (2) provides a qualitatively correct description of the elongational flow of polymer solutions and follows from various structural models; the equations adopted in [4, 5] are identical to Eq. (2) in the particular case when $\alpha = 1$. Using the kinematic

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integral [7] found from the continuity equation, Eq. (2) is solved for $T_{rr}^{(1)}$, $T_{\phi\phi}^{(1)}$, and S is determined

$$S = S^{(1)} + S^{(2)}, \quad S^{(2)} = -4\eta(1-\beta)\dot{R}R^{-1}, \tag{3}$$

$$S^{(1)} \equiv J_\alpha = -\frac{2\beta\eta}{\alpha\lambda R^{4\alpha}(t)} \int_0^t e^{\frac{\xi-t}{\lambda}} (v^{2\alpha} - v^{2\alpha})(v-v)^{-1} \quad R^{2(1-\alpha)}(\xi)\dot{R}(\xi)\,d\xi, \quad v = R^3(\xi), \quad v = R^3(t). \tag{4}$$

Equations (1), (3), and (4) determine the integrodifferential dynamic equation of a cavity in a polymer medium, which is equivalent, when $\alpha = 1$, to the analogous equations of [4, 5]. Note that the integral $J_\alpha(t)$ characterizing the influence of the non-Newtonian part of the stress tensor in Eq. (1) may be determined from a system of first-order differential equations in the case where the integrand may be represented in the form $\sum_k \varphi_k(t)\psi_k(\xi)$. In particular, this is the case with boundary values of the parameter α from the range of variation $\alpha = 1/2$ and $\alpha = 1$, for which, respectively, the following equations equivalent to Eq. (4) are obtained

$$\dot{J}_{1/2} + (\lambda^{-1} + 2\dot{R}R^{-1})J_{1/2} = -4\beta\eta\lambda^{-1}\dot{R}R^{-1}, \tag{5}$$

$$J_1 = J_{11} + J_{12}, \quad \dot{J}_{11} + (\lambda^{-1} + 4\dot{R}R^{-1})J_{11} = -2\beta\eta\lambda^{-1}\dot{R}R^{-1}, \tag{6}$$

$$\dot{J}_{12} + (\lambda^{-1} + \dot{R}R^{-1})J_{12} = -2\beta\eta\lambda^{-1}\dot{R}R^{-1}.$$

For $\alpha = 1/2$, it follows from Eqs. (2) and (5) that $J_{1/2}(t) \equiv T_{rr}^{(1)}(0, t)$. This means that, when $\alpha = 1/2$, rheological properties of the polymer medium with nonlinear pulsations of the cavity that are analogous to conventional viscosity for a Newtonian liquid only appear in the region adjacent to the phase interface and may be taken into account in the boundary conditions in deriving the equation for R(t) [11]. Note in connection with this that with small bubble pulsations the possibility of taking rheological features of a polymer liquid into account solely in terms of the boundary conditions at the surface of the inclusion was rigorously proven in [12]. For $\alpha = 1$, $J_1(t) < T_{rr}^{(1)}(0, t)$, and taking account of the rheology of the medium solely in terms of the boundary condition leads to some slowing of the compression and acceleration of the expansion of the inclusion in comparison with the solution of the accurate system in Eqs. (1), (3), and (6).

For a Newtonian liquid with the viscosity of the polymer solution, $S \equiv S_p = -4n\dot{R}R^{-1}$ in Eq. (1). With closure of the cavity for a state of equilibrium at time $t = 0$, the inequalities $\dot{R} < 0$, $\ddot{R} < 0$ hold in the initial section, and it follows from Eqs. (3) and (4) that $S < S_p$, i.e., taking the relaxational properties of the medium into account leads to acceleration of compression in comparison with the analogous Newtonian liquid [11, 13]. Here $S > S^{(2)}$ always holds when $\beta > 0$, and hence $R(t) > R_s(t)$, where $R_s(t)$ is the law of cavity closure in a pure solvent [5]. With cavity growth, there is a result of analogous significance: $R_p(t) < R(t) < R_s(t)$, where $R_p(t)$ is the law of cavity growth in a Newtonian liquid with the viscosity of the solution [4].

To investigate the nonlinear stage of the process, the dynamic equations of the cavity with $\alpha = 1/2$, 1 are reduced to dimensionless form, assuming, for the sake of simplicity, that $p_g - p_\infty = \text{const}$. It is found that

$$x\ddot{x} + \frac{3}{2}\dot{x} - k + 2\sigma^* x^{-1} + (1-\beta)(\mathrm{Re}_p x)^{-1}\dot{x} - J_\alpha^* = 0, \tag{7}$$

$$\dot{J}_{1/2}^* + \lambda_*^{-1}(1 + 2\lambda_*\dot{x}x^{-1})J_{1/2}^* = -\beta(\mathrm{Re}_p\lambda_*)^{-1}\dot{x}x^{-1}, \tag{8}$$

$$J_1^* = J_{11}^* + J_{12}^*, \quad \dot{J}_{11}^* + \lambda_*^{-1}(1 + 4\lambda_*\dot{x}x^{-1})J_{11}^* = \tag{9}$$

$$= -\frac{1}{2}\beta(\mathrm{Re}_p\lambda_*)^{-1}\dot{x}x^{-1}, \quad \dot{J}_{12}^* + \lambda_*^{-1}(1 + \lambda_*\dot{x}x^{-1})J_{12}^* = -\frac{1}{2}\beta(\mathrm{Re}_p\lambda_*)^{-1}\dot{x}x^{-1},$$

$$k = \operatorname{sign}(p_g - p_\infty), \quad x = R/R_0, \quad R_0 = R(0), \quad \tau = t/t_0,$$

$$J_\alpha^* = J_\alpha (k(p_g - p_\infty))^{-1}, \quad t_p = 4\eta (k(p_g - p_\infty))^{-1},$$

$$t_0 = R_0 (\rho/k(p_g - p_\infty))^{1/2}, \quad \mathrm{Re}_p = t_0/t_p, \quad \lambda_* = \lambda/t_0, \quad \sigma^* = \sigma/(R_0 k(p_g - p_\infty)).$$

In the course of cavity closure, the effective viscosity η_e of the polymer solution in conditions of elongational flow may increase by two or three orders of magnitude. If the corresponding Reynolds number $\mathrm{Re} = \eta/\eta_e \mathrm{Re}_p$ is small, the inertial terms in Eq. (7) may be neglected. For solutions of high-molecular compounds, for example, polyoxyethylene in water [6], the inequalities $\mathrm{Re} \ll \mathrm{Re}_p$ and $\mathrm{Re} < 1$ may be satisfied in extension even if $\mathrm{Re}_p \gg 1$. The system in Eqs. (7)-(9) admits of accurate solution in the inertialess approximation when $2\sigma^* x^{-1} \ll 1$, which corresponds to $R \gg 10^{-6}$ m for $p_\infty \sim 10^5$ Pa. In this case, it follows from Eqs. (7) and (8), for example ($\alpha = 1/2$), that

$$\dot{z} + 2(z - z_1)(z - z_2) = 0, \quad z = \dot{x}x^{-1}, \tag{10}$$

$$z_{1,2} = -\frac{1}{4} A \pm (A^2/16 + B/2)^{1/2}, \tag{11}$$

$$A = \lambda_*^{-1}(1 - \beta)^{-1}(1 - 2k\lambda_* \mathrm{Re}_p), \quad B = k\mathrm{Re}_p \lambda_*^{-1}(1 - \beta)^{-1}.$$

Analysis of Eq. (10) on the phase plane with $k = -1$ shows that $z \to z_1$ as $\tau \to \infty$, if the value $z_0 = z(0)$ determining the initial stress of the Maxwellian element in Eq. (2) satisfies the condition $z_0 > z_2$. Here $z \to z_1 + 0$ if $z_0 > z_1$, and $z \to z_1 - 0$ if $z_2 < z_0 < z_1$. The rest point $z = z_2$ is unstable. When $z_0 < z_2$, closure of the cavity occurs in a finite time. It is important to emphasize that the rate of closure of the cavity $z = z_1$ in asymptotic conditions satisfies the inequality $z_p \leq z_1 \leq 0$, where $z_p = \mathrm{Re}_p$ determines the rate of closure of the inclusion in a viscous liquid with the viscosity of the solution η.

Assuming that the relaxation time λ_* is a small or large parameter, approximate expressions for $z_{1,2}$ may be obtained from Eq. (11)

$$\lambda_* \ll 1 : z_1 = z_p + 2\beta\lambda_* \mathrm{Re}_p^2, \quad z_2 = -[2\lambda_*(1 - \beta)]^{-1},$$

$$\lambda_* \gg 1 : z_1 = -(2\lambda_*)^{-1}, \quad z_2 = -(1 - \beta)^{-1}\left[\mathrm{Re}_p + \frac{1}{2}\beta\lambda_*^{-1}\right].$$

Thus, compression of the cavity in the asymptotic stage occurs more slowly than in a viscous liquid with the viscosity of the solution, which agrees with numerical results [11] and the available experimental data on bubble closure in polymer solutions. Conversely, cavity expansion occurs more rapidly than in an analogous Newtonian liquid: when $k = 1$, it follows from Eq. (11) that $z_p \leq z_1 \leq z_s$, where $z_p = \mathrm{Re}_p$, and $z_s = \mathrm{Re}_p(1 - \beta)^{-1}$ is the asymptotic growth rate of the cavity in pure solvent with viscosity $(1 - \beta)\eta$. In this case, $z_1 = z_p + 2\beta\lambda_* \mathrm{Re}_p^2$ when $\lambda_* \ll 1$ and $z_1 = z_s - 1/2\beta\lambda_*^{-1}(1 - \beta)^{-1}$ when $\lambda_* \gg 1$. Note that the criterion of applicability of the approximate solution in Eq. (11) in cavity growth, as in compression, is the condition $\mathrm{Re} \ll 1$.

The result obtained is explained by the different behavior of the component of the stress tensor $T_{rr}^{(1)}$, determining, as shown above, the influence of the rheology of the medium on the cavity dynamics, in flows of liquid extension and compression, respectively [6]: in the first case, $|T_{rr}^{(1)}|$ may reach significantly higher values than in the second. Analogous results follow from Eqs. (7) and (9) for $\alpha = 1$.

With the aim of investigating transient processes in cavity closure the accurate system in Eqs. (7) and (9) is solved numerically on a computer with $x(0) = 1$, $\dot{x}(0) = J^*(0) = 0$. Characteristic curves are shown in Fig. 1, together with curves of $x_s(\tau)$, $x_p(\tau)$ corresponding

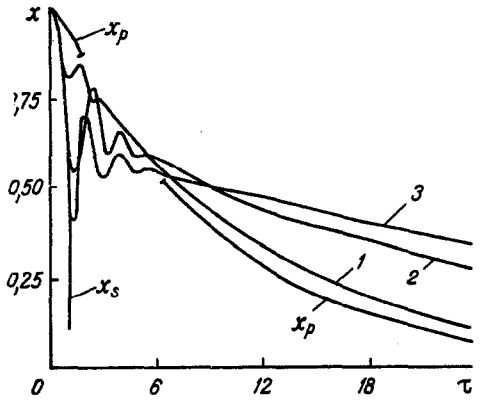

Fig. 1. Dynamics of cavity closure in relaxing liquid ($\sigma^* = 0.005$, $\beta = 0.95$, $Re_p = 0.1$): 1, 2, 3) $\lambda_* = 1, 5, 10$.

to pure solvent and Newtonian liquid with the viscosity of the solution, for comparison. It is evident that, in contrast to Newtonian liquid, closure of the cavity in the polymer solution is nonmonotonic in character: at a definite stage of the process, compression of the bubble stops and expansion begins, on account of the pressure difference which acts. The presence of an oscillatory section on the curve of $x = x(\tau)$ is due to rheological nonlinearity. The initial stages of the process, as noted above, are realized according to a law close to the dependence $x_s(\tau)$. However, the normal stress which develops in the liquid close to the cavity surface in this case rapidly leads to slowing of the rate of compression, and then to change in sign of $\dot{x}(\tau)$. Expansion of the cavity occurs until the stress relaxes to the corresponding value as a result of change in sign of the rate of longitudinal deformation. After damping of the oscillations, the system reaches the noninertial asymptote in accordance with the analytical solution of Eqs. (7) and (9), analogous to Eq. (11). It follows from Fig. 1 and the calculations that increase in relaxation time when Re_p = const increases the length of the oscillatory section and the amplitude of the pulsations; the rate of closure of the cavity at the asymptotic stage decreases simultaneously. Calculations with $\alpha = 1/2$ show that decrease in α is due to more rapid damping of the oscillations, and the radius of the cavity is smaller on reaching the asymptote, which is completely explained by the reduction in level of normal stress in the liquid on account of reduction in the role of nonlinear terms in Eq. (2). Note, in conclusion, that some acceleration in the rate of cavity closure in the concluding stages of the process according to the calculation results in comparison with the asymptote is associated with the appearance of surface tension forces.

NOTATION

$R(t)$, radius of inclusion; ρ, σ, density and surface tension of liquid; t, time; $T_{rr}(y, t)$, $T_{\phi\phi}(y, t)$, components of excess-stress tensor; r, θ, ϕ, spherical coordinate system; $y = 1/3(r^3 - R^3)$, Lagrangian coordinate; p_∞, p_g, pressure at infinity and inside bubble; D/Dt, Jaumann derivative; D, deformation-rate tensor; λ, relaxation time; η, Newtonian viscosity of polymer solution; $0 \leq \beta \leq 1$, measure of the contribution of the dissolved polymer to the effective viscosity of the medium; α, parameter of model.

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VORTEX MOTION IN DILUTE POLYMER SOLUTIONS

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Vortex-dynamic equations have been derived and applied to two problems concerning annular vortices.

1. It has been found that an important part is played by the behavior of individual vortex structures in turbulent transport [1, 2] from measurements on turbulent boundary layers and friction reduction by polymers (about 0.001-0.01%). Here I present a quasi-one-dimensional approach to vortex dynamics (vortex filaments of finite thickness) in dilute polymer solutions.

Measurements show that a major stage (vortex stretching) can be retarded by the entropy-dependent elasticity in macromolecular coils. The vortex is formed in a boundary layer or by a generator via velocity pulsations or pulses arising from the generator. The vorticity is generated in a shear layer at the boundary of the flow, with the leading part acquiring a mushroom form and being transferred to the core of the vortex [2]. At the head of the flow, the liquid particles are spread out as occurs in a free disk film arising from the collision of a thin jet with a small target [3]. One therefore expects that the vortex will contain macromolecular coils at its core, which are stretched along the axis. If the vortex is then stretched at a rate exceeding θ^{-1}, the longitudinal elastic stresses in the core will increase (otherwise, they will decrease). These stresses influence the velocity pattern and thus the core evolution. The vortex is surrounded by unstretched liquid, and although the liquid and the core contain macromolecules, the elastic stresses in the latter are negligible, and it may be considered as ideal (if vortex diffusion is negligible).

We now consider a vortex whose core is subject to a longitudinal elastic stress (the liquid is considered as incompressible). We assume as a first approximation that this stress is constant over the cross section of the core. Correspondingly, the stress tensor in the core will be $\sigma' = \sigma'_{\tau\tau}\,\tau\tau$. The sum of this elastic tensor σ' and the viscous-stress tensor is the deviator for the stress tensor in the liquid [4]. The equation for the core vorticity is

$$\frac{D\Omega}{Dt} = (\Omega\cdot\nabla)\,\mathbf{v} + \nu\Delta\Omega + \frac{1}{\rho}\nabla\times(\nabla\cdot\sigma'). \tag{1}$$

From (1), viscous effects are unimportant for times less than $t < m^2/\nu$, which we consider.

The contribution from the elastic stresses in (1) is

$$\nabla\times(\nabla\cdot\sigma') = \mathbf{b}\left(\sigma'_{\tau\tau}\frac{\partial k_*}{\partial\xi} + 2k_*\frac{\partial\sigma'_{\tau\tau}}{\partial\xi}\right) - k_*\varkappa_*\sigma'_{\tau\tau}\mathbf{n}. \tag{2}$$

The right side of (2) is zero for rectilinear and annular vortices and is small in the long-wave approximation for any vortex. We therefore restrict ourselves to that approximation for times $t < m^2/\nu$, where the second and third terms on the right in (1) can be

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